

BELL TELEPHONE LABORATORIES
INCORPORATED

OCTOBER
1929



REPRINT
B-428

BARKHAUSEN EFFECT IN IRON,
NICKEL AND PERMALLOY--I

BY

RICHARD M. BOZORTH

MEASUREMENTS SHOWING THAT UNDER CERTAIN CONDITIONS
THE MAGNETIC DISCONTINUITIES ACCOUNT FOR THE
WHOLE CHANGE IN MAGNETIZATION

REPRINT OF PAPER

Abstract Presented before

THE AMERICAN PHYSICAL SOCIETY
NEW YORK, N. Y. FEBRUARY, 1929

Published in

THE PHYSICAL REVIEW
SECOND SERIES, VOL. 34, PP. 772-784 SEPTEMBER 1, 1929

BARKHAUSEN EFFECT IN IRON, NICKEL AND PERMALLOY. I. MEASUREMENT OF DISCONTINUOUS CHANGE IN MAGNETIZATION

BY RICHARD M. BOZORTH
BELL TELEPHONE LABORATORIES, NEW YORK

(Received July 24, 1929)

ABSTRACT

These experiments show that the changes in magnetization which take place suddenly in large groups of atoms account quantitatively for the whole change in magnetization which corresponds to the steeper part of the hysteresis loop. Since it is estimated that discontinuities corresponding to groups of 10^{10} atoms (10^{-13} cm³) or less will not appreciably affect the measuring system, it is concluded that the magnetic moments of only a very small number of atoms are reversed in groups smaller than this. The maximum size of the discontinuities for all materials examined, independent of crystal size, is found to be about the same, corresponding to the complete reversals of the elementary magnets contained in a volume of about 10^{-6} cm³. The discordance of the results of previous measurements by others is explained as due to the different rates of decay of eddy-currents in the samples used.

INTRODUCTION

EXPERIMENTS have shown that the Barkhausen effect is due to discontinuities in magnetization,¹ and that the larger discontinuities correspond to the reversal of as many magnetic elements as are contained in a volume² of 10^{-5} or 10^{-6} cm³. But it has not been known whether the observed discontinuities account for any considerable fraction of the total change in induction, or whether this change can best be accounted for by a very large number of discontinuities most of which correspond to the simultaneous reversals of but a few atoms. It has been proposed³ that each single crystal reverses its magnetic moment at once, and on this view one would expect that in annealed materials the whole change in induction could be accounted for by discontinuities large enough to observe. The size of the larger discontinuities also suggested to early observers that each crystal in the metal changed its magnetization as a unit. On the other hand, Tyndall² found that the size of the discontinuities did not depend on the grain size in silicon steel, and recently Sizoo⁴ has come to a similar conclusion for electrolytic iron. The observation has frequently been made that there is a larger effect in hard-worked than annealed materials, from which it might be inferred that the groups are as large, or larger in material with very small crystals than they are in the comparatively coarse grained annealed materials.

¹ H. Barkhausen, *Phys. Zeits.* **20**, 401-403 (1919); *Zeits. Tech. Phys.* **5**, 518-519 (1924) B. van der Pol, *Proc. Acad. Sci. Amsterdam* **23**, 637-643, 980-988 (1920).

² B. van der Pol, reference 1; E. P. T. Tyndall, *Phys. Rev.* **24**, 439-451 (1924).

³ P. Weiss and G. Ribaud, *Journal de physique* **3**, 74-80 (1922).

⁴ G. J. Sizoo, *Physica* **9**, 43-50 (1929).

Not only are there these different possibilities as to the average size of the discontinuities, but there is also a great variety of experimental data⁵ for the Barkhausen effect observed as the noise in a telephone receiver or the current output from an amplifier, in various materials having different shapes and sizes and permeabilities. The lack of an explanation of these differences in the effect, and the uncertainty as to the average size of the discontinuities, made it desirable to investigate these questions further.

It is the purpose of this paper to describe experiments which show that on the steep part of the hysteresis loop practically the whole change in induction takes place in steps large enough to detect with an amplifier in the usual way, and only a very small part of the change in induction occurs in steps corresponding to the simultaneous action of 10^{10} or fewer atoms.⁶ The different estimates of the magnitude of the Barkhausen effect observed as the noise in a telephone receiver, or as the current output from an amplifier, are seen to depend on the different rates of decay of eddy-currents in the materials of different sizes and permeabilities. A following paper⁷ will describe the determination of the volume of the material which corresponds to one discontinuous change in magnetization of average size, assuming that this change is from saturation in one direction to saturation in the other.

THE EXPERIMENTAL PROCEDURE

Principle of the method. The magnetic material to be examined is placed in a slowly and uniformly changing magnetic field. When a discontinuous change in the induction B occurs, an e.m.f. is created in a coil of wire surrounding the sample. This e.m.f. is proportional to dB/dt , and rises quickly to a maximum and then decreases to zero as illustrated in Fig. 16. The area under the e.m.f.-vs.-time curve is proportional to the change in B which occurred as a single Barkhausen impulse. A great many such single impulses occur during a complete magnetic cycle, perhaps a million per cm^3 , and the problem is to integrate the areas under the dB/dt -vs.-time (or e.m.f.-vs.-time) curves. This is done by amplifying the e.m.f.'s, and arranging the amplifier output so that the current measured is the average value, over a period of several seconds, of currents which are at all times proportional to dB/dt . This average current, multiplied by the time, gives the whole change in B which occurs during this time in discontinuities large enough for the apparatus to detect.

Description of apparatus. The apparatus used is shown diagrammatically in Fig. 1. A slowly changing magnetic field is produced in the magnetizing coil M by means of the apparatus shown at the left. Here a condenser C

⁵ W. Gerlach, E. T. Z. **42**, 1293 (1921); W. Gerlach and P. Lertes, Zeits. f. Physik **4**, 383-392 (1921), Phys. Zeits. **22**, 568-569 (1921); K. Zschiesche, Zeits. f. Physik **11**, 201-214 (1922); S. Procopiu, Comptes rendus **184**, 1163-1165 (1927); E. P. T. Tyndall and J. M. B. Kellogg, Phys. Rev. **30**, 354-356 (1927); J. Pfaffenberger, Ann. d. Physik **87**, 737-768 (1928).

⁶ A preliminary report of this work was given at the New York Meeting of the American Physical Society, February 23, 1929, Phys. Rev. **33**, 636-7 (1929).

⁷ A preliminary report was given at the Washington Meeting of the American Physical Society, April 18, 1929, Phys. Rev. **33**, 1071 (1929).

is slowly charged by a small current flowing through a vacuum tube V_1 . The potential on the condenser controls the grid bias of another vacuum tube V_2 , the space current of which flows in the windings of the coil. A battery and resistance in parallel with the coil permit the strength of the magnetic field to be changed between -10 and $+10$ gauss at a rate proportional to the current charging the condenser C . By shorting this condenser the process may be repeated. The rate of change of field can easily be changed from 0.001 to 10 gauss/sec. by changing a resistance R_1 in the filament circuit of V_1 . The magnetizing coil surrounds the sample and the two search coils S_1 and S_2 , which are wound on spools 2.5 cm long and 2.5 cm in outside diameter and are connected to the amplifier in series opposition. The amplifier has four stages and is resistance-capacity-coupled with $200,000$ ohms and 1 microfarad, and has one-half megohm grid leaks. Its performance was tested up to $10,000$ cycles per second at which point its efficiency was found to have decreased but slightly. The output current from the amplifier is passed through a resistance R_2 of about $50,000$ ohms, and the constant potential produced in this, when there is a constant voltage on the input, is neutralized by a battery B_1 . In series with this battery and resistance are

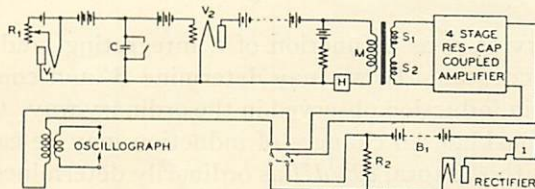


Fig. 1. Diagram of apparatus.

a rectifier⁸ and a meter which registers the current. On account of the high resistance R_2 and the characteristics of the rectifier the relation between the input voltage on the amplifier and the current read on the meter is approximately linear. The battery in series with the rectifier is adjusted until the current indicated by the meter is 10 microamperes. When a variable alternating voltage is applied to the first stage of the amplifier the output current is very nearly proportional to the average value of this voltage for currents ranging from 20 to 300 microamperes. The range usually used is from 20 to 200 microamperes.

The whole apparatus is shielded by enclosing it in iron boxes electrically connected together. The first stage vacuum tube of the amplifier and the tube through which the magnetizing current passes are suspended by rubber bands in small lead boxes mounted on springs, to protect them from mechanical vibrations.

Operation. When a sudden change in magnetization, characteristic of the Barkhausen effect, occurs in the sample inside of one of the search coils, the electromotive force which acts on the amplifier is proportional to dB/dt , the rate of change of induction in the sample, which is determined by the

⁸ Western Electric Vacuum Tube 244-A with grid connected to plate.

rate of decay of the eddy-currents. For calculating dB/dt for a spontaneous change in magnetization we may use the expression given by Rayleigh⁹ or that given by Wwedensky.¹⁰ According to the early calculation of Rayleigh the change in induction takes place in accordance with the equation:

$$B = B_0 \left[1 - \exp \left(- \frac{1.44 \times 10^9 \rho t}{\pi r^2 \mu} \right) \right]$$

where B_0 is the total change in induction, ρ the resistivity in ohm-cm, and μ the permeability, in this case the reversible permeability or permeability for small alternating forces. Wwedensky's calculation indicated that the initial rate of change of induction is infinite but drops rapidly to the value given by Rayleigh, following his equation after $B/B_0 = 0.1$.

If the apparatus is distortionless the output current i is proportional to dB/dt . If the meter reads i_1 , the average of the instantaneous values of i , the total change in induction which takes place discontinuously may be calculated from the equation

$$B' = A_1 \int i_1 dt,$$

so that by observing i_1 as a function of t , integrating, and multiplying by the appropriate constant A_1 , we may determine B' and compare it with B , the total change in induction observed in the ordinary way. Or, more simply, dB'/dH due to Barkhausen changes of induction may be calculated from i_1 and compared with the total dB/dH as ordinarily determined.

Although it has been shown that eddy-currents decay approximately according to the equation of Rayleigh given above, nevertheless it should be emphasized that dB'/dH may be calculated from i_1 quite independently of the validity of that equation.

When a sudden change in induction occurs in the second search coil, surrounding another portion of the sample, an electromotive force is produced in the opposite direction to that produced in the first coil, but the rectifier prevents current so produced from passing through the meter. The purpose of the second coil is two-fold, first to neutralize the effect of any stray alternating magnetic field which penetrates the two shields of 1/4 inch copper and 3/8 inch annealed permalloy surrounding the magnetizing coil; and second to make the mean voltage applied to the amplifier equal to zero so that the charges on the coupling condensers produced by many successive impulses in one direction will not be lost by discharging through the grid leaks. A test was made to ensure that the same sudden change in induction did not affect both search coils. The coils were placed various distances apart and i_1 observed for a certain change in induction for each separation. It was found that i_1 increased as the separation increased until the separation was 4 or 5 cm, after which i_1 became practically constant. Accordingly measurements were made with the coils about 7 cm apart.

⁹ Rayleigh, Rep. Brit. Assn. 52, 446-447 (1882).

¹⁰ B. Wwedensky, Ann. d. Physik 64, 609-620 (1921).

When the times between successive voltage impulses, caused by successive discontinuities, are short enough the voltage impulses from one search coil will overlap those from the other and the current i_1 will be too small. Consequently, in order to measure the whole discontinuous change in induction the field must be changed very slowly, and in reality it is necessary to make measurements for various speeds and extrapolate to zero speed.

When making measurements the strength of the magnetic field is changed at a convenient rate and the output current i_1 noted for various values of the current through the solenoid as indicated by the meter H . The rate of change of field strength is determined at the same time, using a stop-watch. For most samples in the form of wires 1 mm in diameter and 20 cm long, i_1 is noted each time the field-strength changes by 0.1 gauss. Although the current i_1 fluctuates considerably between readings the average value can usually be determined with considerable accuracy. For any given value of the field strength, dB'/dH is obtained from i_1 , dH/dt , and a constant determined by calibrating the apparatus.

Calibration. The apparatus is calibrated by removing the sample and placing coaxially with one of the search coils a long single layer coil through which is passed a small measured sinusoidal current of known frequency. Complete flux linkage between this calibrating coil and one of the search coils, and the absence of flux linkage with the other search coils, were ensured. The current through the calibrating coil produces a calculable mean rate of change of flux in the place occupied later by the sample, and it is assumed that the same rate of change of flux will produce a current i_1 whether that change is produced in a coil containing air or in a sample of ferromagnetic material.

Denoting by A the ratio of the mean rate of change of flux in maxwells per second, to the output current i_1 in microamperes,

$$\frac{d\phi}{dt} = A i_1,$$

and

$$A = 0.8\pi 2^{\frac{1}{2}} n f A_c \alpha$$

where f is the frequency of the current, n is the number of turns of wire per cm on the calibrating coil, A_c is its cross-sectional area, and α is the ratio of the r.m.s. current in amperes in the coil to the output current i_1 which flows at the same time. Then

$$\frac{dB'}{dt} = \frac{A i_1}{S}$$

where S is the cross-sectional area of the sample. For the apparatus used, $A = 3.40 \times 10^{-3}$ maxwells/second/microampere. In the experiments described in this paper $S = 0.0081$ cm², so that $dB'/dH = 0.420 i_1 dH/dt$.

A sample calibration curve is shown in Fig. 2.

THE EXPERIMENTAL RESULTS

The following materials were examined:

Armco iron, hard drawn.

Armco iron, vacuum annealed 3 hrs. at 1100°C , 1 hr. at 880°C , cooled $300^{\circ}/\text{hr}$.

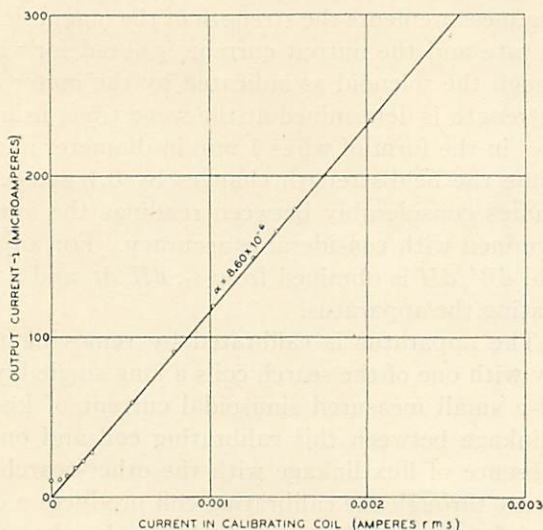


Fig. 2. Typical calibration curve.

Permalloy, 81 percent Ni, hard drawn from 3 mm diameter.

Permalloy, 81 percent Ni, vacuum annealed 3 hrs. at 1100°C , cooled $300^{\circ}/\text{hr}$.

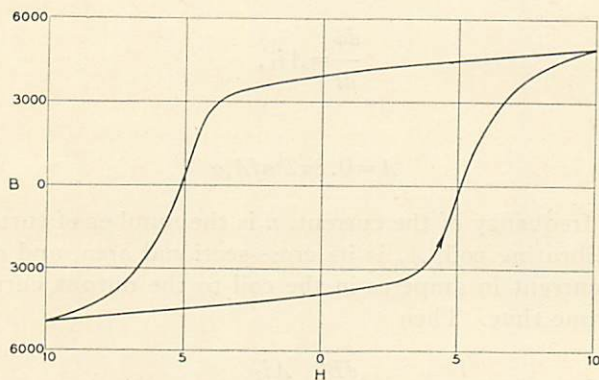


Fig. 3. Hysteresis loop for hard drawn permalloy wire composed of 81 percent Ni and 19 percent Fe.

Permalloy, 50 percent Ni, vacuum annealed 3 hrs. at 1100°C , cooled $300^{\circ}/\text{hr}$.

Nickel, vacuum annealed 3 hrs. at 1100°C , cooled $300^{\circ}/\text{hr}$.

The specimens in each case were 1 mm in diameter and 20 cm long.

The data for hard permalloy are shown in Figs. 3 and 4. Fig. 3 is the hysteresis loop and Fig. 4(a) indicates the slope of the lower branch of the loop for various values of H . These curves were both taken with a ballistic

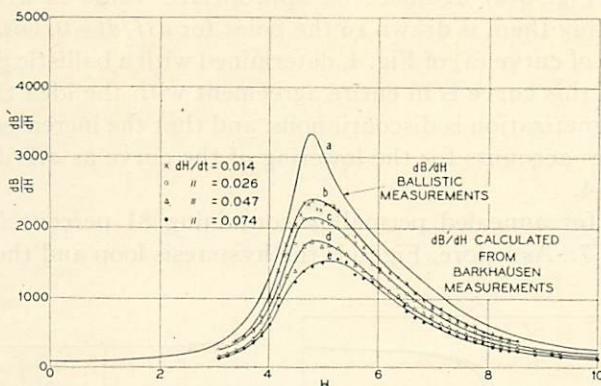


Fig. 4. (a) represents the slope of the lower branch of the hysteresis loop of Fig. 3. The other curves show dB/dH calculated from Barkhausen measurements using different time rates of change of magnetic field.

galvanometer, using the same coils in the same positions as for the measurements of the Barkhausen effect, the only difference being that the leads from one of the coils were reversed so that they were aiding instead of opposing

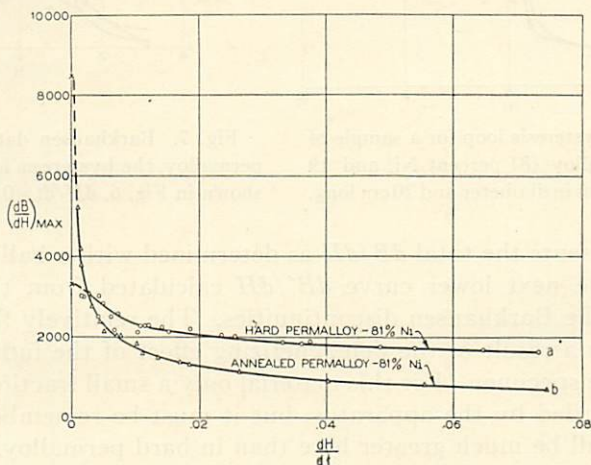


Fig. 5. This shows that when the field-strength is changed more and more slowly the peaks of curves such as (b) to (e) in Fig. 4, approach the peak of curve (a) in Fig. 4. Similar data are shown for annealed permalloy of the same chemical composition.

each other. Curves (b), (c), (d) and (e) represent for various values of H the values of dB'/dH calculated as described above from the output current i_1 . These four curves are taken with various rates of change of field-strength as shown in the figure, and do not coincide because there are differ-

ent amounts of overlapping of the impulse in the two coils, as already described under "Operation." But even when the overlapping is considerable, over half of the total change in flux is accounted for by the measured discontinuities. The largest values of dB'/dH for various values of dH/dt , are plotted ¹¹ in Fig. 5(a) against the appropriate value of dH/dt , and the curve connecting them is drawn to the point for $dH/dt=0$, corresponding to the maximum of curve (a) of Fig. 4, determined with a ballistic galvanometer. The course of this curve is in entire agreement with the idea that the whole change in magnetization is discontinuous, and that the increased overlapping of the impulses accounts for the lowering of the curve as dH/dt increases as shown in Fig. 4.

The data for annealed permalloy containing 81 percent Ni are shown in Figs. 6 and 7. As before, Fig. 6 is the hysteresis loop and the upper curve

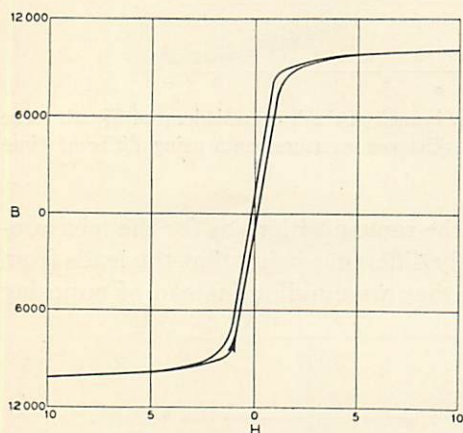


Fig. 6. Hysteresis loop for a sample of annealed permalloy (81 percent Ni, and 19 percent Fe) 1 mm in diameter and 20 cm long.

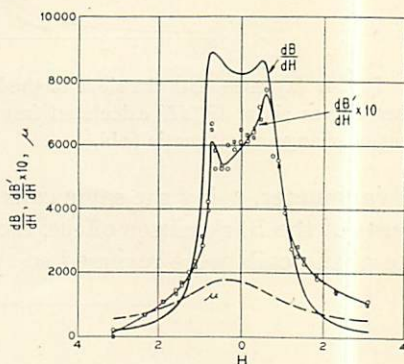


Fig. 7. Barkhausen data for annealed permalloy, the hysteresis loop for which is shown in Fig. 6, $dH/dt=0.047$ gauss/sec.

in Fig. 7 represents the total dB/dH as determined with a ballistic galvanometer, and the next lower curve dB'/dH calculated from the current i_1 produced by the Barkhausen discontinuities. The relatively flat top of the upper curve is a result of the demagnetizing effect of the induced poles at the ends of the specimen. For this material only a small fraction of the total dB/dH is recorded by the apparatus, but it must be remembered that the overlapping will be much greater here than in hard permalloy. The reason for this is the slower decay of eddy-currents due to the greater reversible permeability, which is about 2000 in annealed permalloy as compared with 80 in hard permalloy. The maximum values of the calculated dB'/dH , for different rates of change in field-strength, are shown in Fig. 5(b).

¹¹ For this series of measurements the calibration was made so that the output current was 4 microamperes when the current in the calibrating coil was zero, instead of 10 microamperes as shown in Fig. 2.

The dB'/dH curve in Fig. 7 differs from the total dB/dH curve in shape as well as height. This seems to be due to two causes: the center of gravity of the permeability curve, shown as a dotted line in the figure, is displaced towards lower (or more negative) fields with respect to that of the total dB/dH curve, causing an increase in the calculated dB'/dH towards the right because of less overlapping due to smaller permeabilities; and an increase in the average size of the discontinuities for higher fields, resulting also in increased calculated dB'/dH for higher fields because of less overlapping. For example, if the average size of the discontinuities is twice as great in one case as in another, the current for one impulse of the first average size will be twice as great as for the second for corresponding parts of the

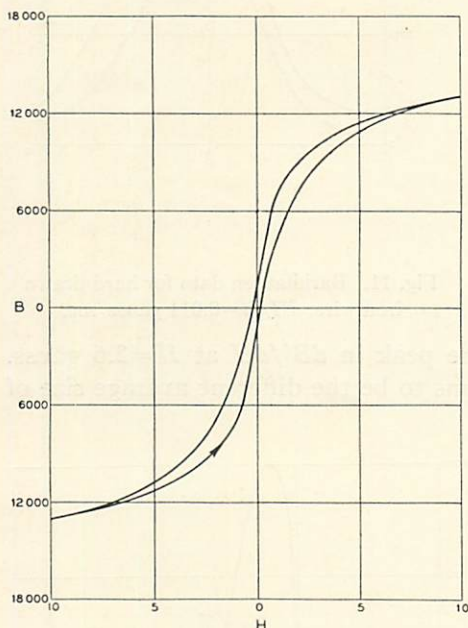


Fig. 8. Hysteresis loop for annealed permalloy containing 50 percent Ni and 50 percent Fe.

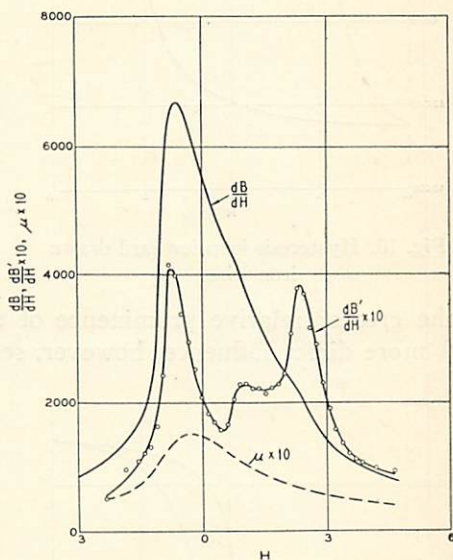


Fig. 9. Barkhausen data for permalloy containing 50 percent Ni and 50 percent Fe, $dH/dt = 0.046$ gauss/sec.

impulse, but the time constant for the two cases will be the same. The chance of overlapping of two impulses in opposite directions will be one-fourth as great in the first case as the second because the total number of impulses of each kind is one-half as great, but for each time that overlapping occurs the amount of overlapping will be twice as great. On the average, then, the loss by overlapping will be less for the larger impulses and the ratio of calculated dB'/dH to total dB/dH will be greater. Other experiments show that the largest discontinuities for 81 permalloy also occur where the calculated dB'/dH is a maximum.

Data for annealed permalloy containing 50 percent Ni are shown in Figs. 8 and 9. As shown in the latter figure the maximum of the μ -vs- H curve

is displaced with respect to the maximum of the ballistic dB/dH -vs- H curve in the direction opposite to that for 81 Ni permalloy. This helps to explain

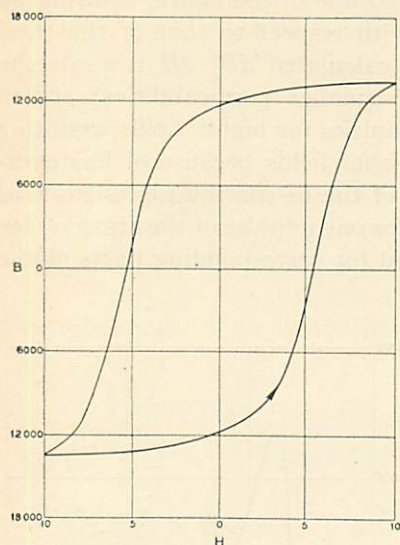


Fig. 10. Hysteresis loop for hard drawn iron wire.

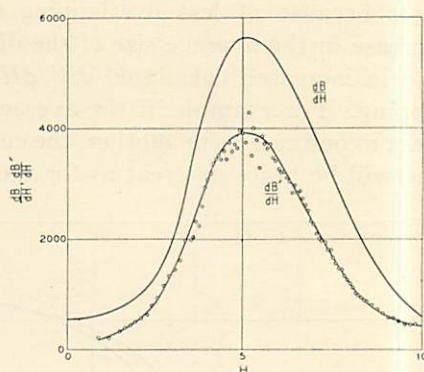


Fig. 11. Barkhausen data for hard drawn iron wire, $dH/dt=0.011$ gauss/sec.

the greater relative prominence of the peak in dB'/dH at $H=2.6$ gauss. A more direct influence, however, seems to be the different average size of

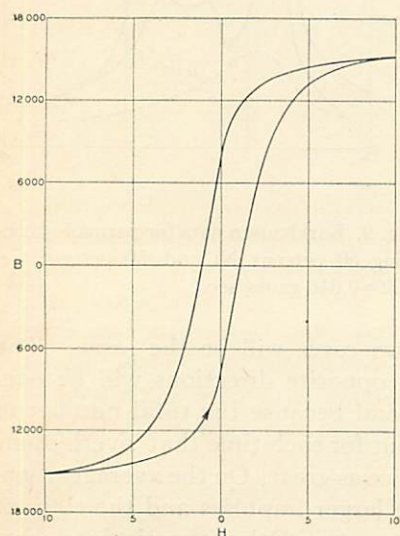


Fig. 12. Hysteresis loop for annealed iron.

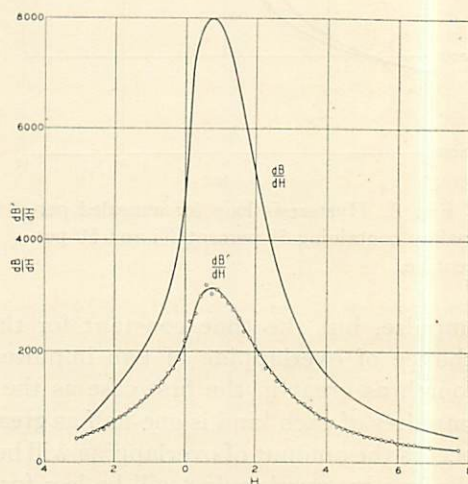


Fig. 13. Barkhausen data for annealed iron, $dH/dt=0.032$ gauss/sec.

the discontinuities at the two peaks. Experiments to be reported later show that the average size for this specimen is a maximum at about 2.6 gauss,

corresponding to the last maximum of dB'/dH shown in Fig. 9. The two maxima in the dB'/dH curve as well as the constricted "wasp-waisted" hysteresis loop of Fig. 8 suggest an inhomogeneous¹² magnetic structure, and suggest also the possibility of learning more about the nature of such inhomogeneities by measurements of the Barkhausen effect.

Data for hard and annealed Fe and for annealed Ni are shown in Figs. 10 to 15. For hard Ni the fields available were not as great as the coercive force, so quantitative measurements were not made.

To illustrate the decay of eddy-currents in hard and annealed permalloy, and the effect of overlapping of impulses produced in the two search coils, oscillograph records were made, as shown in Fig. 16. The output of the amplifier was passed through the primary of a transformer, the secondary of which was connected to a Westinghouse oscillograph, type N. The upper trace is for a 2 mm wire of well annealed permalloy containing 78 percent Ni. Here the decay of eddy-currents is relatively slow on account of the large

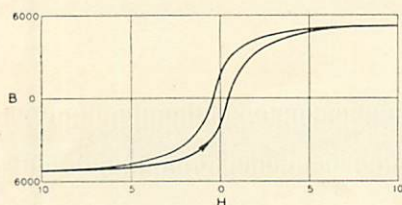


Fig. 14. Hysteresis loop for annealed nickel.

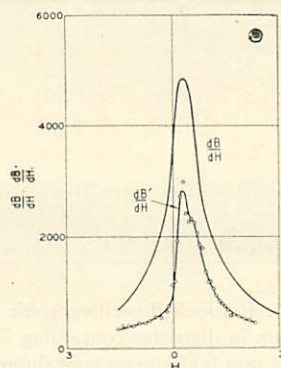


Fig. 15. Barkhausen data for annealed nickel, $dH/dt=0.012$ gauss/sec.

diameter and high permeability. This should be contrasted with the middle trace which shows the effect in hard-drawn permalloy 1 mm in diameter. In the latter case the decay is so rapid that the oscillograph does not record properly the form of the curve; nevertheless, the area under the curve is a true measure of the size of the discontinuity in magnetization, and can be compared with the area under the impulses in the upper trace. The volumes of material corresponding to the separate impulses have been calculated in a few cases for these specimens and also for other materials in both the hard drawn and annealed conditions, and found to be about 10^{-6} cm³ for the larger impulses in all cases. This volume is about the same as that observed by Tyndall for silicon steel. The rate of decay of eddy-currents as determined from the upper trace and the 1000 cycle timing wave given in the lower trace, may be compared with the rate calculated from Wwedensky's

¹² E. Gumlich, Arch. f. Electrot. 9, 153-166 (1920); G. Elmen, J. Franklin Inst. 206, 317-338 (1928).

formula given above. The calculated time constant for this sample is 0.0015 sec., and agrees tolerably well with that estimated from the oscillographic record.

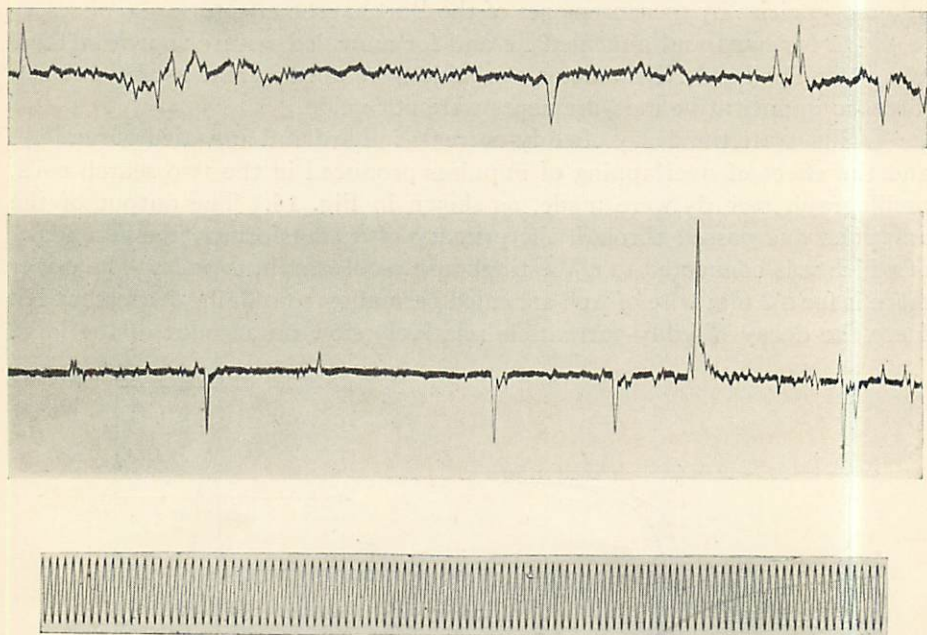


Fig. 16. Retouched oscillographic records of Barkhausen effect. Upper trace, record for sample 2 mm in diameter containing 78 percent Ni and 22 percent Fe. Middle trace, record for sample 1 mm in diameter containing 81 percent Ni and 19 percent Fe. Lower trace, timing line of frequency 1000 cycles per second.

DISCUSSION OF THE RESULTS

These experiments show that the whole change in magnetization on the steeper parts of the hysteresis loops of hard worked samples of iron and permalloy is accounted for by the magnetic discontinuities detected and measured by the apparatus. The larger discontinuities correspond to the complete reversal of magnetization of about 10^{17} atoms, or a volume of 10^{-6} cm³. Since it is estimated that discontinuities corresponding to groups of 10^{10} atoms will not appreciably affect the measuring system, it is concluded that a very small proportion of the discontinuities correspond to groups containing less than this number of atoms. In other words, the Barkhausen discontinuities which account for practically the whole change in magnetization are due to the simultaneous action of a great many atoms and the magnetization of only a relatively small number of atoms is reversed in groups of less than 10^{10} . This conclusion applies, of course, only to changes in magnetization on the steeper parts of the hysteresis loop. Although for annealed permalloy the rate of change of magnetization calculated from

Barkhausen measurements is never more than about two-thirds of the total rate of change, this would be expected because the reversible permeability of the material is very high and the loss due to overlapping is correspondingly great. The most reasonable conclusion is that in all of the materials examined the number of magnetic changes involving less than 10^{10} atoms is very small on the steeper parts of the hysteresis loops. This conclusion is supported by other evidence of a different kind to be reported later.

Experiments on the Barkhausen effect have previously been made with various materials and with several kinds of apparatus, with a great variety of results. These results can now be explained as due to the different rates of decay of eddy-currents in these materials of different sizes and shapes and permeabilities. For example, the difference in the intensity of the sound in the telephone receivers when annealed permalloy and hard drawn permalloy specimens are compared, is very marked. Because of the more rapid decay of the eddy-currents in the latter, the instantaneous current flowing through the receivers is much greater for this material than for soft permalloy when the size of the discontinuity in magnetization, proportional to the area under the current-*vs*-time curve, is the same for each. Consequently the receiver diaphragm will move with greater speed and amplitude and the noise will be many times greater for hard worked permalloy with a reversible permeability of 80 than for annealed permalloy with a reversible permeability of 2000. This great difference cannot be attributed to different sizes of discontinuities because the larger impulses on oscillographic records are about the same for each. A similar difference in the character of the sound is observed for thick and thin wires of the same material having similar hysteresis loops. In the experiment using fine wires a sharp crackling sound is observed in contrast to a rustling sound observed using thicker wires. This is because the decay of eddy-currents is more rapid in the fine wires and the output current from the amplifier is correspondingly greater and fluctuates more rapidly.

The general parallelism between the Barkhausen effect and hysteresis¹³ is apparently due to the character of the eddy-current decay, for when the hysteresis is low the reversible permeability is generally high and the Barkhausen effect as measured by ear is consequently low.

I take pleasure in acknowledging the benefit I have derived from technical discussions with Dr. O. E. Buckley, and from the assistance of Mr. Joy F. Dillinger with the apparatus.

¹³ W. Gerlach and P. Lertes, and K. Zschiesche, reference 5.

BELL TELEPHONE LABORATORIES
INCORPORATED
463 WEST STREET, NEW YORK